Physics 513, Quantum Field Theory

Homework 7

Due Tuesday, 4^{th} November 2003

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Symmetry Factors

Throughout the following derivations it will be helpful to state explicitly a method to obtain the symmetry factor for a given diagram. The method is derived from the published lecture notes of Professor Colin Morningstar of Carnegie Mellon University.[1]

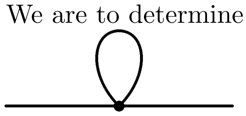
The *symmetry factor* of a given diagram is given by

$$S = \frac{n!(\eta)^n}{r},$$

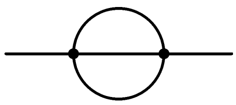
where n is number of vertices, η is a coupling constant, and r is the multiplicity of the diagram. The value of η is 4! in ϕ^4-theory and 3! in Yukawa theory. This pattern implies η will be 4 for question 1(b) below.

To determine the multiplicity r, all external points are labelled and all vertices are drawn with four (or three) lines emerging. All of these lines are assumed to be distinguishable. The total number of ways to connect the external points and vertices to form the diagram equals the multiplicity r. If a diagram is direction sensitive, then this is taken into account by only including the number of ways to draw the diagram given the directional conditions onf the external points.

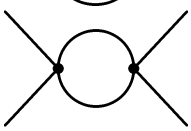
1. a) We are to determine the symmetry factor for four diagrams.



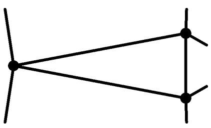
$r = 4 \cdot 3 = 12$ and $n = 1$ so $S = \frac{1!(4!)^1}{12} = 2.$



$r = 8 \cdot 4 \cdot 3 \cdot 2 = 192$ and $n = 2$ so $S = \frac{2!(4!)^2}{192} = 6.$

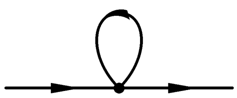


$r = 8 \cdot 3 \cdot 4 \cdot 3 \cdot 2 = 576$ and $n = 2$ so $S = \frac{2!(4!)^2}{576} = 2.$

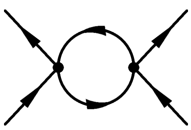


$r = 12 \cdot 3 \cdot 8 \cdot 3 \cdot 4 \cdot 3 \cdot 4 \cdot 2 = 82944$ and $n = 3$ so $S = \frac{3!(4!)^3}{82944} = 1.$

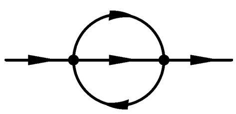
b) We are to determine the symmetry factors for the following diagrams.



$r = 2 \cdot 2 = 4$ and $n = 1$ so $S = \frac{1!(4)^1}{4} = 1.$



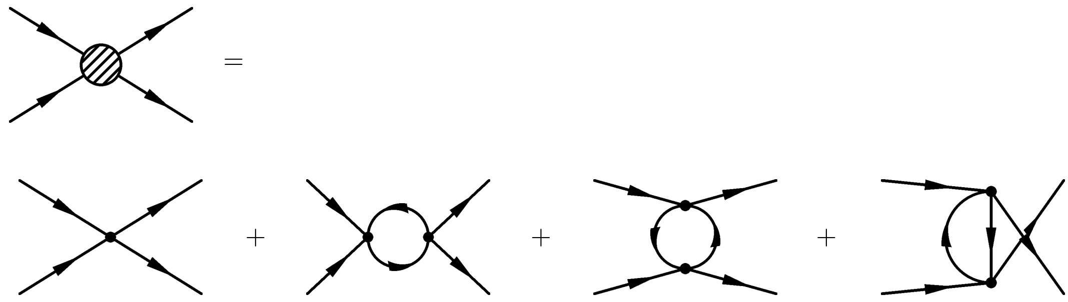
$r = 4 \cdot 2 \cdot 2 \cdot 2 = 32$ and $n = 2$ so $S = \frac{2!(4)^2}{32} = 1.$



$r = 4 \cdot 2 \cdot 2 = 16$ and $n = 2$ so $S = \frac{2!(4)^2}{16} = 2.$

[1] Chapter 9, page 141. Available at http://www.andrew.cmu.edu/course/33-770/.

2. a) We are asked to draw all distinct Feynman diagrams for the four point function of the ϕ^4-theory given below to the order λ^2.



i **ii** **iii** **iv**

b) We are to calculate the contributions from each diagram. Note that I have included explicit symmetry conservation in the above diagrams. For example, for contribution (ii), I have made the substitutions $k_1 = k$ and $k_2 = k - p_1 - p_2$; I have made similar substitutions for the other diagrams as well. Thus, including symmetry factors, the contributions are,

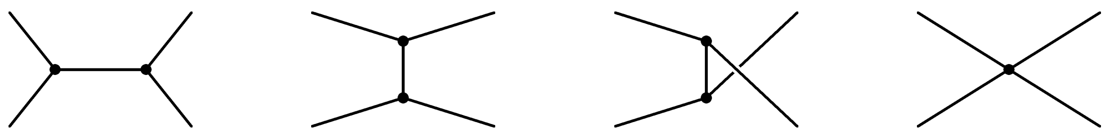
i) $(-i\lambda)(2\pi)^4\delta^{(4)}(p_3 + p_4 - p_1 - p_2)$;

ii) $\dfrac{(-i\lambda)^2}{2}\displaystyle\int \frac{d^4k}{(2\pi)^4}(2\pi)^4\delta^{(4)}(p_3 + p_4 - p_1 - p_2)\frac{i}{(k^2 - m^2 + i\epsilon)}\frac{i}{((k - p_1 - p_2)^2 - m^2 + i\epsilon)}$;

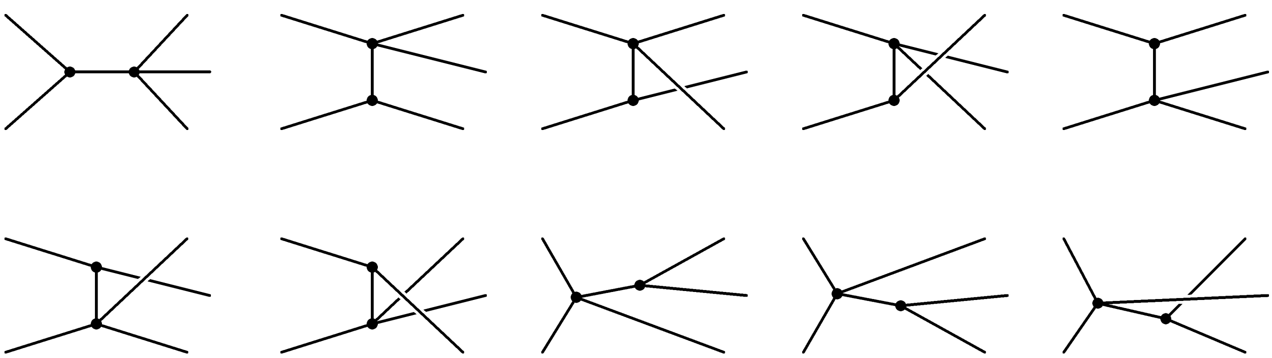
iii) $\dfrac{(-i\lambda)^2}{2}\displaystyle\int \frac{d^4k}{(2\pi)^4}(2\pi)^4\delta^{(4)}(p_3 + p_4 - p_1 - p_2)\frac{i}{(k^2 - m^2 + i\epsilon)}\frac{i}{((k + p_1 - p_3)^2 - m^2 + i\epsilon)}$;

iv) $\dfrac{(-i\lambda)^2}{2}\displaystyle\int \frac{d^4k}{(2\pi)^4}(2\pi)^4\delta^{(4)}(p_3 + p_4 - p_1 - p_2)\frac{i}{(k^2 - m^2 + i\epsilon)}\frac{i}{((k + p_1 - p_4)^2 - m^2 + i\epsilon)}$.

3. a) We are to draw all of the Feynman diagrams up to order λ or g^2 for the scattering process $p_A + p_B \to p_a + p_3$. These are given below.



b) Like part (a) above, we are to draw all of the Feynman diagrams of order $g\lambda$ for the process $p_A + p_B \to p_1 + p_2 + p_3$. Note that the labels are implied after the first diagram on the left of each row. There are 10, and they are given below.



c) It is clear that all of the symmetry factors are 1. I have directly computed them, but it is unnecessary to repeat those trivial calculations here. Rather, it is enough to notice that there are no loops in any of the diagrams. Each vertex connects unique, distinguishable fields. This is equivalent to the observation that the topology of each diagram above was enough to specify it entirely. Therefore, all symmetry factors are 1.